

## ALGEBRA 1 HONORS STANDARDS

**IMPORTANT NOTE:** The following 4 topics are NOT to be used at any time on an Algebra 1 test.

1. Complex numbers
2. Sum/Difference of cubes
3. Trigonometry
4. Probability/Statistics

*Any topics NOT on this list CANNOT be included on any Algebra I test.*

***Compare and contrast the real number system and its various subsystems with regard to their structural characteristics.***

- Identify and apply the field properties of Real number system and its subsystems.
- Identify subsystems of Real numbers.
- Identify and apply the properties of equality and order to the Real number system and its subsystems.
- Apply the definition of absolute value in algebraic and geometric situations.

***Demonstrate an understanding of algebraic procedures and symbolism.***

- Translate between word phrases/sentences and algebraic expressions/equations and/or inequalities.
- Perform operations on Real numbers.
- Perform operations on polynomials.
- Factor polynomials. (see above for restrictions)
- Use the laws of exponents.
- Solve linear equations and inequalities in one variable.
- Solve a system of two first-degree equations with two variables using a variety of strategies.
- Solve absolute value equations.

***Use algebraic and related strategies to solve problems.***

- Solve real world and mathematical problems using first-degree equations and inequalities.
- Solve real world and mathematical problems using a system of two first-degree equations in two variables.
- Solve the following types of word problems: mixture, percent, work, distance/rate/time, coin, and current/wind speed.

***Demonstrate an understanding of the geometry associated with equations and inequalities.***

- Determine and apply relationships among a first-degree equation in one variable; its corresponding inequalities; and their number line graphs.
- Determine and apply relationships among a simple first-degree equation in one variable involving absolute value; its corresponding inequalities; and their number line graphs.
- Determine relationships involving numerical coefficients; slopes; and y-intercept; between first-degree equations in two variables and lines in the Cartesian plane.
- Determine the relationships between first-degree inequalities in two variables and half-plane in the Cartesian plane.
- Determine solutions in systems of two first-degree equations in two variables using graphs in the Cartesian coordinate system.
- Compare the solutions of a quadratic equation in one variable with the x-intercepts of the Cartesian graph of the corresponding quadratic function.

**ONLY THE ABOVE CAN BE INCLUDED ON JANUARY AND FEBRUARY TESTS**  
**THE FOLLOWING TOPICS MAY BE INCLUDED ON MARCH TESTS ONLY.**

- Perform operations on rational algebraic expressions.
- Perform operations of radical expressions.
- Pythagorean Theorem, Midpoint formula, Distance formula
- Solve quadratic equations in one variable.
- Solve equations involving rational algebraic expressions and radical expressions.
- Solve equations involving absolute value.
- Represent and interpret functions and relations with ordered pairs; mapping tables; and Cartesian graphs.
- Use and apply functional notation in situations involving mappings; tables; and Cartesian graphs.
- Solve real world and mathematical problems using equations with rational or radical expressions in one variable.
- Solve real world and mathematical problems involving direct and inverse variation.

## Geometry Standards

### January, February and March Topics

#### I. Points, Lines, Planes and Angles

- a) points, lines, planes
- b) naming lines, segments, rays, finding midpoint, finding distance
- c) naming and finding measures of angles
- d) postulates and theorems relating points, lines and planes
- e) use the terms collinear, coplanar, intersection

#### II. Deductive Reasoning

- a) if-then statements, converses, inverses, contrapositives
- b) special pairs of angles – vertical, complementary, supplementary
- c) perpendicular lines

#### III. Parallel Lines and Planes

- a) definitions, skew lines
- b) properties of parallel lines
- c) proving lines parallel
- d) theorems involving parallel lines
- e) angles of a triangle
- f) angles of a polygon, number of diagonals

#### IV. Congruent Triangles

- a) proving triangles congruent
- b) using congruent triangles (CPCTC)
- c) the isosceles triangle theorems
- d) medians, altitudes, and perpendicular bisectors

#### V. Quadrilaterals

- a) properties of parallelograms
- b) ways to prove that quadrilaterals are parallelograms
- c) theorems involving parallel lines
- d) special parallelograms
- e) trapezoids
- f) kites

#### VI. Inequalities in Geometry

- a) indirect proofs
- b) inequalities for one triangle
- c) inequalities for two triangles

#### VII. Similar Polygons

- a) ratio and proportion
- b) properties of proportion
- c) similar polygons
- d) proving triangles similar
- e) proportional lengths

#### VIII. Right Triangles

- a) similarity in right triangles (geometric mean)

- b) the Pythagorean Theorem
- c) converse of the Pythagorean Theorem
- d) special right triangles
- e) sine, cosine and trig ratios
- f) applications of right triangle trig

## **February and March Topics**

### **I. Circles**

- a) basic terms
- b) tangents
- c) arcs and central angles
- d) arcs and chords
- e) inscribed angles
- f) angles with vertex in the circle and outside the circle
- g) circles and lengths of segments

### **II. Constructions and Loci**

- a) perpendiculars and parallels
- b) concurrent lines
- c) the meaning of locus, locus problems, locus and construction

### **III. Areas of Plane Figures**

- a) areas of rectangles, parallelograms, triangles, rhombuses, trapezoids, kites, regular polygons, equilateral triangles, squares
- b) circumferences and areas of circles
- c) arc lengths and sectors of circles
- d) ratios of areas

## **March Topics**

### **I. Lateral Area, Total Area and Volume of Solids**

- a) Prisms
- b) Pyramids
- c) Cylinders
- d) Cones
- e) Spheres
- f) Areas and volumes of similar solids

## Algebra 2 Standards

1. Real numbers and properties
  - order of operations
2. Equations and inequalities
  - literal equations and formulas
  - conjunctions and disjunctions
  - compound sentences with inequalities
  - absolute value equations and inequalities
3. Linear equations and inequalities
  - relations
  - graphing equations
  - functions
  - slope
  - parallel
  - perpendicular
  - equations of a line
  - direct variation
  - inverse functions
4. Systems
  - systems of equations
  - systems of inequalities
  - determinants
  - Cramer's rule
5. Radicals and irrational numbers
  - power functions
  - roots and radicals
  - rational and irrational numbers
  - operations with radicals
  - complex numbers
6. Polynomial functions and rational expressions
  - solving quadratic and higher order polynomial equations
  - discriminant
  - polynomial functions and their graphs
  - fractional equations
  - quadratic inequalities and their graphs
  - remainder and factor theorems
  - The Fundamental Theorem of Algebra
  - rational functions
7. Exponents and Logarithms
  - rational exponents
  - real exponents
  - exponential functions
  - logarithmic functions
  - properties of logarithms
8. Conics

- circles
- parabolas
- ellipses
- hyperbolas
- graphing and solving quadratic systems
- inverse variation

### **End of standards for January and February**

#### 9. Sequences and Series

- arithmetic and geometric sequences
- arithmetic and geometric series
- infinite geometric series

#### 10. Permutations, combinations and probability

- fundamental counting principle
- linear and circular permutations
- counting subsets
- combinations and products
- mutually exclusive and independent events
- binomial expansion
- binomial theorem
- Pascal's triangle

#### 11. Matrices

- basic properties
- operations
- linear systems

## PRE-CALCULUS STANDARDS

*Topics 1 – 8 are appropriate for January contests*

**Topics 1 – 11 are appropriate for February contests**

**Topics 1 – 14 are appropriate for contests in March and April**

**For any month, only the standards as listed may be used.**

**1. Demonstrate an understanding of the theory of functions.**

- find domains; ranges; an specific values of functions in functional notation.
- given two functions perform the algebra of functions including composition of functions.
- determine if a given function is:
  - a. symmetric (with respect to the axes and/or origin.
  - b. periodic
  - c. monotonic
  - d. bounded
  - e. continuous
- identify and graph polynomial and rational functions and determine asymptotes.
- define and use parametric forms of functions and convert from parametric to Cartesian form.
- given a function; determine the inverse and state whether or not the inverse is a function.

**2. Demonstrate an understanding of connection between circular and trigonometric functions and their inverses.**

- evaluate circular and trigonometric expressions involving any of the six functions and their inverses.
- given the equation for a circular (trigonometric) function; identify and/or sketch the graph and the graph of its inverse relation and state the domain and range of the original function and its associated inverse function.
- identify its equation when given a graph of any of the six circular functions.
- state the period; amplitude; phase shift; and vertical shift of a circular function and/or graph of the function.

**3. Demonstrate an understanding of the trigonometric identities.**

- prove that a given trigonometric equation is an identity by applying the Pythagorean relation and reciprocal identities.
- prove that an appropriate trigonometric equation is an identity when given the sum and difference formulas for the cosine; sine; and tangent.
- prove that an appropriate trigonometric equation is an identity when given the double order formulas for sine; cosine; and tangent.
- prove that an appropriate trigonometric equation is an identity when given the half-angle formulas for sine; cosine; and tangent.

**4. Demonstrate the ability to apply trigonometry to problem solving situations.**

- solve a right triangle given two sides; or a side and an acute angle.
- use the appropriate trigonometric function(s) to solve problems involving right or oblique triangles.
- apply the Law of Sines.
- apply the Law of Cosines.
- find the area of an oblique triangle.
- estimate the solution to a problem involving a right or oblique triangle.
- in the SSA case determine whether 0; 1; or 2 triangles exist and determine the

- triangles (if they exist)

**5. Demonstrate the ability to solve a variety of trigonometric (circular) equations.**

- find the general solutions to a trigonometric equation
- find particular solutions to a trigonometric equation within a given domain.
- solve equations involving inverse of circular/trigonometric functions.

**6. Demonstrate an understanding of conic sections and loci.**

- given the description of a locus determine the equation of the locus.
- given the equation of a line determine slope and y-intercept; and graph the line.
- given the equation of a circle determine the center and radius; and graph it.
- given the equation of a parabola determine vertex; focus; and directrix; and graph it.
- given equation of an ellipse in standard form; determine the center; foci; and vertices; graph it.
- given the equation of a hyperbola in standard form; determine the foci; vertices; and asymptotes; and graph it.
- determine new equations resulting from translation or rotation of axes.
- identify the graph of any second degree equation.
- express a quadratic equation in general form  $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$  and use  $B^2 - 4AC$  to distinguish conics.
- recognize degenerate and imaginary cases.

**7. Demonstrate an understanding of the relationship between exponential and logarithmic functions and their application to problem situations.**

- evaluate expressions involving rational exponents.
- sketch the graphs of exponential functions and logarithmic functions of different bases.
- solve equations involving exponential functions and logarithmic functions.
- solve real-world problems involving exponential functions and logarithmic functions.
- simplify expressions using the relationships between logarithms and exponents.
- express the number e and the expression 'e to the x' as infinite series

**8. Demonstrate the ability to solve problems using concepts from matrix algebra.**

- apply determinants to solve systems of equations.
- invert a square matrix.

**\*\*\*\*\*END OF STANDARDS FOR JANUARY\*\*\*\*\***

**9. Demonstrate the ability to solve problems using vectors.**

- find a vector in standard position equal to a given vector.
- determine magnitude and direction of vectors.
- identify perpendicular and parallel vectors.
- determine the measure of the angle between two vectors.
- resolve a vector into component vectors.
- add and subtract vectors and multiply a vector by a scalar.
- find the dot product of two vectors.
- use vectors to solve real world problems.

**10. Demonstrate an understanding of polynomial and rational functions; their parametric equations and their graphs.**

- given a polynomial function determine intercepts and sketch the graph.



- given an equation of rational function determine intercepts and asymptotes and sketch the graph.
- given a set of parametric equations sketch the graph.

**11. Demonstrate an understanding of graphs in the polar coordinate system and their relation to the Cartesian coordinate system.**

- graph points in the polar coordinate system.
- convert between polar coordinates and Cartesian coordinates.
- express complex numbers in polar or trigonometric form.
- convert equations in polar form to Cartesian form.
- convert equations in Cartesian form to polar form.
- graph polar equations and identify specific types (roses; limacons; spirals; and conics)
- use de Moivre's theorem to find powers and roots of complex numbers.

\*\*\*\*\***END OF STANDARDS FOR FEBRUARY**\*\*\*\*\*

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**12. Demonstrate understanding of mathematical induction and sequences and series.**

- given an expression of rule for the nth term find any term of the sequence.
- given a sequence find a formula for the nth term in the sequence.
- find the nth term of a binomial expansion.
- find the sum of an arithmetic series.
- find the sum of a finite or infinite geometric series if it exists.
- define convergent and divergent sequences and series, determine limits if they exist.
- determine whether a sequence is increasing or decreasing.
- find the least upper bound and greatest lower bound of a sequence if they exist.
- express a series in sigma notation.
- use mathematical induction to prove series formulas.
- use mathematical induction to prove inequality formulas.

**13. Demonstrate the ability to solve problems using probability and statistics.**

- find probabilities of simple events.
- find probabilities using venn diagrams.
- find probabilities of mutually exclusive events.
- find probabilities of independent events.
- define an event and/or the complement of an event.
- find probabilities of the complement of an event.
- find conditional probabilities.
- find probabilities in binomial distributions.
- determine a standard (z) score in a normal distribution.

**14. Demonstrate an understanding of the concept of limits and its applications.**

- geometrically illustrate functions for which  $x$  increases without bound and find limits, if they exist.
- find when possible for any neighborhood of a number  $L$ ; a neighborhood of a point  $a$  such that  $f(x)$  is in the neighborhood of  $L$  when  $x$  is in the neighborhood of  $a$ .
- calculate limits of functions using theorems about limits.
- geometrically illustrate functions which are continuous at a point and/or continuous on an interval.
- given a rational function  $f(x)$  find the limit if it exists at a point of discontinuity.
- using the definition of the derivative of  $f(x)$  find the derivative function.
- determine the equation of tangents to graphs of curves given the slope formula.

**END OF STANDARDS FOR MARCH**

# **FAMAT Calculus Standards – June 1, 2014**

## **Distribution of Topics**

**January Tests:** At least 75% of topics from Sections 2 to 9 and no more than three questions with topics from Sections 10 to 12. Questions containing topics from Section 1 should also contain a topic from another section, and should be of at least Moderate difficulty.

**February Tests:** At least 75% of topics from Sections 2 to 11 and no more than three questions with topics from Sections 12 and 13. Questions containing topics from Section 1 should also contain a topic from another section, and should be of at least Moderate difficulty. Questions containing topics from Section 13 should be of Moderate-Hard or Hard difficulty.

**March Tests:** At least 75% of topics from Sections 2 to 12, and no more than four questions with topics from Section 13. Questions containing topics from Section 1 should also contain a topic from another section, and should be of at least Moderate difficulty. Questions containing topics from Section 13 should be of at least Moderate difficulty.

### ***1. Standard Algebra and Precalculus Concepts***

1.01 Functions, Conics, and Polynomials

1.02 Trigonometry

1.03 Matrices & Vectors

1.04 Non-Calculus Sequences & Series

1.05 Probability and Counting

### ***2. Demonstrate the ability to apply the concept of limits to functions.***

2.01 An intuitive understanding of the limiting process

2.02 Calculating limits using algebra

2.03 Estimating limits from graphs or tables of data

2.04 Limit definition of  $e$ .

### ***3. Demonstrate the ability to identify asymptotic and unbounded behavior.***

3.01 Understanding asymptotes in terms of graphical behavior

3.02 Describing asymptotic behavior in terms of limits involving infinity

3.03 Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)

### ***4. Demonstrate an understanding of continuity.***

4.01 An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)

4.02 Understanding continuity in terms of limits

4.03 Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)

### ***5. Apply the concept of the derivative.***

5.01 Derivative presented graphically, numerically, and analytically

5.02 Derivative interpreted as an instantaneous rate of change

5.03 Derivative defined as the limit of the difference quotient

5.04 Relationship between differentiability and continuity

### ***6. Demonstrate the ability to apply derivatives to find the slope of a curve and tangent and normal lines to a curve and as an instantaneous rate of change.***

6.01 Slope of a curve at a point.

6.02 Tangent line to a curve at a point and local linear approximation

6.03 Normal line to a curve at a point

6.04 Instantaneous rate of change as the limit of average rate of change

6.05 Approximate rate of change from graphs and tables of values

**7. Demonstrate the ability to compute derivatives of algebraic; trigonometric; exponential; and logarithmic functions.**

7.01 Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions

7.02 Derivative rules for sums, products, and quotients of functions

7.03 Chain rule and implicit differentiation

7.04 Find the derivative of the inverse of a function

7.05 Find higher order derivatives

**8. Demonstrate the ability to identify increasing and decreasing functions; relative and absolute maximum and minimum points; concavity; and points of inflection.**

8.01 Corresponding characteristics of graphs of  $f$  and  $f'$

8.02 Relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$

8.03 The Mean Value Theorem and its geometric interpretation

8.04 Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

8.05 Corresponding characteristics of the graphs of  $f$ ,  $f'$ , and  $f''$

8.06 Relationship between the concavity of  $f$  and the sign of  $f''$

8.07 Points of inflection as places where concavity changes

**9. Demonstrate an understanding of applications of the derivative.**

9.01 Analysis of curves, including the notions of monotonicity and concavity

9.02 Optimization, both absolute (global) and relative (local) extrema

9.03 Modeling rates of change, including related rates problems

9.04 Use of implicit differentiation to find the derivative of an inverse function

9.05 Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

**10. Demonstrate the ability to interpret definite integrals and use properties of definite integrals**

10.01 Basic properties of definite integrals (examples include additivity and linearity)

10.02 Numerical approximations to definite integrals. Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values

10.03 Definite integral as a limit of Riemann sums

10.04 Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

**11. Demonstrate the ability to use the techniques of integration**

11.01 Use of the Fundamental Theorem to evaluate definite integrals

11.02 Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined

11.03 Antiderivatives following directly from derivatives of basic functions, including algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions

11.04 Antiderivatives by substitution of variables (including change of limits for definite integrals)

**12. Demonstrate the ability to apply antiderivatives to solve problems including growth and decay, particle motion, and finding areas and volumes**

12.01 Finding specific antiderivatives using initial conditions, including applications to motion along a line

12.02 Solving separable differential equations and using them in modeling

12.03 Find the average value of a function

12.04 Find the total distance traveled by a particle along a line

12.05 Find the accumulated change from a given rate of change

12.06 Find the area under a curve using integration

- 12.07 Find the volume of solids of revolution using disc, washer, or shell methods
- 12.08 Find the volume of solids with known cross sections
- 12.09 Use integration appropriately to model physical, biological, or economic situations or other similar applications

### **13. Calculus BC Topics**

- 13.01 L'Hospital's Rule
- 13.02 Derivatives of parametric, polar, and vector functions
- 13.03 Area of regions bounded by polar curves
- 13.04 Numerical solution of differential equations using Euler's method
- 13.05 Antiderivatives by parts and by partial fractions
- 13.06 Improper integrals (as limits of definite integrals)
- 13.07 Series convergence tests, including those related to the harmonic series, alternating series with error bound, integral and p-tests, ratio and root tests, and comparison tests
- 13.08 Functions defined by power series
- 13.09 Radius and Intervals of convergence of power series
- 13.10 Taylor's theorem and the Maclaurin series of basic functions (exponential, sine, cosine,  $1/(1-x)$ )
- 13.11 Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.

## **Distribution of Problem Difficulty for All Calculus Tests**

Problem difficulty will be gauged on the approximate percentage of students that would get a particular question correct over a Combined Regional testing. It should be noted that Statewide competition percentages correct tend to be higher than Combined Regional percentages correct, so problem difficulty may be slightly higher for Statewide competitions in order to maintain the below distribution of scores, at the discretion of the editors.

### **For a 30 question test:**

**10 Questions** should be considered **Easy**, with  $\geq 75\%$  of students getting these questions correct in the Combined Regional format.

**5 Questions** should be considered **Moderate-Easy**, with **50-75%** of students getting these questions correct in the Combined Regional format.

**5 Questions** should be considered **Moderate**, with **25-50%** of students getting these questions correct in the Combined Regional format.

**5 Questions** should be considered **Moderate-Hard**, with **5-25%** of students getting these questions correct in the Combined Regional format.

**5 Questions** should be considered **Hard**, with  $\leq 5\%$  of students getting these questions correct in the Combined Regional format.

Please see **Example Problems** below as a guideline for the **difficulty level** of questions, based upon past Combined Regional results.

**Please note that these are ranges of difficulty level, and within each difficulty level problems should be distributed across the range of percentage correct listed above.**

## A Couple of Notes On Problem Solutions:

- (1) A problem that is more difficult requires a more detailed solution. Solutions should skip no steps and should be thought of as a written version of how one would teach how to do the given problem in a classroom.
- (2) Many problems have multiple ways to solve them. Some of those ways may be easier, but outside the Curriculum. Therefore, it is important to note that the solution to a given problem must use only content that is in the Curriculum, and that the difficulty of a problem will be based on that solution, even if advanced knowledge renders a question trivial. Therefore, it is recommended that such questions not be included among the Hard questions of a test, since they will not challenge the most upper-level students effectively.

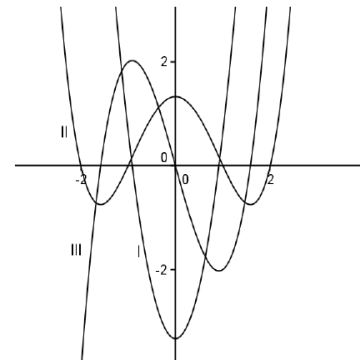
## Example Problems

### 10 Questions: Easy ( $\geq 75\%$ Correct)

**January, 2012 (75.7%)**

25. Three graphs, labeled I, II, and III are displayed in the figure to the right. One of the graphs is  $f(x)$ , one is  $f'(x)$ , and one is  $f''(x)$ . Which of the following identifies each of the three graphs?

	$f(x)$	$f'(x)$	$f''(x)$
A)	I	II	III
B)	II	I	III
C)	III	I	II
D)	II	III	I
E)	NOTA		



**February, 2011 (76.2%)**

17. At what value of  $x$  does the function  $f(x) = x^3 - 3x^2 - 9x$  change from decreasing to increasing?
- A. 3      B. -3      C. -1      D. 1      E. NOTA

**January, 2010 (84.5%)**

- 3) What is the antiderivative of  $\cos(x)$ ?
- A.  $-\cos(x) + C$       B.  $-\sin(x) + C$       C.  $\cos(x) + C$       D.  $\sin(x) + C$       E. NOTA

**February, 2011 (86.4%)**

14. Find  $f'(x)$  if  $f(x) = \sqrt[3]{5x^2 - x + 4}$ .

- A.  $\frac{10x-1}{3\sqrt[3]{(5x^2-x+4)^2}}$     B.  $\frac{10x-1}{2\sqrt[3]{(5x^2-x+4)^2}}$     C.  $\frac{10x}{3\sqrt[3]{(5x^2-x+4)^2}}$     D.  $\frac{10x-1}{3\sqrt[3]{(5x^2-x+4)}}$   
E. NOTA

**February, 2012 (91.9%)**

1) Evaluate  $\lim_{x \rightarrow 5} 2$

- A) -1    B) 0    C) 1    D) 2    E) NOTA

**5 Questions: Moderate – Easy (50-75% Correct)**

**February, 2011 (50.4%)**

2. Find all the points on the curve  $x^2y^2 + xy = 2$ , where the slope of the tangent line is -1.

- A. (-1, -1), (-1, 1)    B. (1, -1), (1, 1)    C. (1, 1), (1, 2)    D. (-1, -1), (1, 1)    E. NOTA

**January, 2012 (57.0%)**

2. Find the slope of the line normal to  $f(x) = x^2 \cdot 2^x$  at  $x=2$ .

- A)  $\frac{-1}{8+8\ln 2}$     B)  $\frac{-1}{16+16\ln 2}$     C)  $\frac{1}{16+16\ln 2}$     D)  $-\frac{1}{32}$     E) NOTA

**March, 2013 (60.4%)**

6. There exists a function such that  $P(x) = (R - C)(x)$ . If  $P(x) = x^2 + 4x + 3$  and  $C(x) = x^2 - 9x + 6$ . What is  $R'(10)$ ?

- A. 143    B. 35    C. 24    D. 9    E. NOTA

**February, 2013 (67.2%)**

4. If  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$  for all  $a \in \mathbb{R}$ , then  $f(x)$  *must* be:

- A. even                      B. odd                      C. both even and odd                      D. neither even nor odd                      E. NOTA

**February, 2009 (73.4%)**

24. Evaluate:  $\int_{-2}^2 (4x^3 - 3x^2 + 2x - 1) dx$ .

- A. -20                      B. -12                      C. 20                      D. 60                      E. NOTA

**5 Questions: Moderate (25-50% Correct)**

**February, 2009 (26.2%)**

13. Grant's Hope Powered Dream Machine outputs dreampower,  $d$ , based on the amount of hope,  $h$ , for the economy as well as the number of supporters in millions,  $s$  (i.e. if 2 million people support him,  $s = 2$ ). The relation is  $d = \frac{(h^2 + 2)^s - 2^s}{(h^2 + 2)^{s-2} - 2^{s-2}}$ . If he has 5 million supporters, as the economy becomes hopeless (i.e.  $h \rightarrow 0$ ) what does the dreampower output approach?

- A. 4                      B.  $\frac{20}{3}$                       C. 8                      D.  $\frac{40}{3}$                       E. NOTA

**February, 2011 (26.6%)**

15. If  $x^y = y^x$ , find  $y'$ .

- A.  $y/x$                       B.  $\frac{x(\ln x^y - y)}{y(\ln x^y - x)}$                       C.  $\frac{y(\ln x^y + y)}{x(\ln x^y - x)}$                       D.  $\frac{y(\ln y^x - y)}{x(\ln x^y - x)}$                       E. NOTA

**March, 2013 (38.9%)**

13.  $\int_{-3}^{\frac{3\pi}{2}} \frac{dx}{x^2 + 6x + 10} =$

- A.  $\arctan\left(\frac{3\pi + 6}{2}\right)$                       B.  $\arctan\left(\frac{3\pi - 6}{2}\right)$                       C.  $\arctan\left(\frac{3\pi - 3}{2}\right)$                       D.  $\arctan\left(\frac{3\pi + 3}{2}\right)$                       E. NOTA

**March, 2013 (41.2%)**

12. For some function  $f(x)$  over an interval  $I$ ,  $f'(x) < 0$ ,  $f''(x) > 0$ . Order the following from least to greatest area:

- |  |            |
|--|------------|
| 1. The left hand Riemann sum approximation.  | A. 1,2,3,4 |
| 2. The Trapezoidal sum approximation         | B. 4,3,2,1 |
| 3. The right hand Riemann sum approximation. | C. 1,2,4,3 |
| 4. The definite integral over $I$ .          | D. 3,4,2,1 |
|  | E. NOTA    |

**March, 2010 (48.4%)**

17. Alex is throwing darts at a rectangular dartboard in the  $x$ - $y$  plane bounded by the lines  $x = 0$ ,  $x = 3$ ,  $y = 0$ , and  $y = 9$ . On this dartboard there is a target, which is the region below the curve  $y = x^2$ . Alex has bad aim and for any two points  $P$  and  $Q$  on the dartboard, the probability that he hits  $P$  is equal to the probability that he hits  $Q$ . If Alex always hits the dartboard, then what is the probability that he hits the target?

- A.  $\frac{1}{3}$                       B.  $\frac{7}{20}$                       C.  $\frac{3}{8}$                       D.  $\frac{1}{2}$                       E. NOTA

**5 Questions: Moderate – Hard (5-25% Correct)**

**March, 2012 (5.9%)**

16. Daniel and Payal are playing a game with  $f(x) = \frac{x-3}{x-2}$ . They take turns reciting the value of  $f^{(n)}(1)$ ; that is, Daniel gives the value of  $f'(1)$ , Payal gives the value of  $f''(1)$ , Daniel gives the value of  $f'''(1)$ ,

and so on. Diego is computing the sequence  $\{s_k\}$  with  $k$ th term  $s_k = \frac{\sum_{i=1}^k \frac{P_i}{D_i}}{k^2}$ , where  $P_i$  is the  $i$ th value Payal gives and  $D_i$  is the  $i$ th value Daniel gives. Compute  $\lim_{k \rightarrow \infty} s_k$ , the value to which  $\{s_k\}$  converges.

- A)  $1/2$                       B) 1                      C)  $\pi^2/6$                       D)  $e$                       E) NOTA



**February, 2011 (6.4%)**

23. Find  $G'(x)$  given  $G(x) = \int_x^1 x^2 \sqrt{u^2 + 1} du$ .

A.  $-x \sqrt{x^2 + 1}$     B.  $-x^2 \sqrt{x^2 + 1}$     C.  $-x^2 \sqrt{x^2 + 1} - 2x \int_x^1 \sqrt{u^2 + 1} du$

D.  $-x^2 \sqrt{x^2 + 1} + 2x \int_x^1 \sqrt{u^2 + 1} du$     E. NOTA

**March, 2013 (13.8%)**

21.  $\frac{dy}{dx} = (xy + x + y + 1)^2$ . If  $y = 0$  when  $x = 2$ , what is  $y$  when  $x = -4$ ?

A.  $-\frac{3}{4}$     B.  $\frac{7}{9}$     C.  $\frac{12}{13}$     D.  $-\frac{18}{19}$     E. NOTA

**February, 2013 (14.0%)**

9. Find the maximum value of  $\sum_{k=1}^n \left( \frac{(-1)^k}{k!} \cdot \int_0^k 3x^2 dx \right)$  where  $n$  is a positive integer.

A. 2    B. 3    C. 4    D.  $\sqrt{e^3}$     E. NOTA

**February, 2009 (22.9%)**

14. Let  $f(x) = \ln(\cos(\operatorname{arccot}(x)))$ . Find  $|f'(\sqrt{3})|$ .

A.  $\frac{\sqrt{3}}{12}$     B.  $\frac{\sqrt{3}}{8}$     C.  $\frac{1}{4}$     D.  $\frac{\sqrt{3}}{4}$     E. NOTA

**5 Questions: Hard (≤ 5% Correct)**

**February, 2009 (5.0%)**

20. Let  $a(x) = x^4 + 3x^2 + 6$ . Let  $b$  be the rate of change of the  $y$ -intercept of the tangent line to  $a(x)$ . Find  $b$  when  $x = 2$ .

A. -108    B. -54    C. 54    D. 108    E. NOTA

**February, 2013 (4.7%)**

12. Find  $f(e) + f\left(\frac{1}{e}\right)$  where  $f(x) = \int_1^x \frac{\ln(t)}{1+t} dt$  for  $x > 0$ . *Hint:* It may be helpful to first calculate, more generally,  $f'(x) + f'\left(\frac{1}{x}\right)$ , and then use that result to find  $f(x) + f\left(\frac{1}{x}\right)$ .
- A.  $\frac{1}{e}$                       B.  $\frac{1}{2}$                       C. 1                      D.  $e$                       E. NOTA

**February, 2013 (3.9%)**

26. When

$$\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$$

is written in the form  $\frac{a\pi^b}{c}$ , where  $a$ ,  $b$ , and  $c$  are positive integers such that  $a$  and  $c$  are relatively prime, what is the value of  $a + b + c$ ? *Hint:* Substitute  $u = \pi - x$ .

- A. 4                      B. 5                      C. 6                      D. 7                      E. NOTA

**March, 2013 (3.2%)**

29.  $\int_2^{2\sqrt{3}} \left( \sqrt{\frac{4}{x^2} + 1} \right) dx$

- a.  $4 + \sqrt{2} - \ln(1 + \sqrt{2})$                       b.  $4 - 2\sqrt{2} + \ln\left(\frac{1 + \sqrt{2}}{3}\right)$
- c.  $4 + \sqrt{2} - \ln\left(1 + \frac{\sqrt{2}}{2}\right)$                       d.  $4 - 2\sqrt{2} + \ln\left(1 + \frac{2\sqrt{2}}{3}\right)$

e. NOTA

**February, 2009 (1.3%)**

30. Evaluate:  $\int_0^{\frac{\pi}{4}} \ln(\tan(x) + 1) dx$ . (*Hint:* Work backwards with angle addition formulas)

- A.  $\frac{\pi \ln 2}{8}$                       B.  $\frac{\pi \ln 2}{4}$                       C.  $\frac{\pi \ln 2}{2}$                       D.  $\pi \ln(2)$                       E. NOTA

Statistics Test Guidelines by Rob Snow, June 2004

I. Exploring Data

A. Interpreting graphs of distributions of one variable data (stemplot, histogram)

1. Center and spread
2. Outliers and unusual features
3. Shape (symmetric, skewed)

B. Summarizing distributions of one variable data

1. Mean and median
2. Range, Interquartile Range, standard deviation
3. Quartiles, percentiles, z-scores

4. Boxplots

5. effect of linear transformations on summary statistics

C. Comparing distributions of one variable data (back to back stemplots, parallel boxplots)

1. Compare center and spread
2. Compare clusters and gaps
3. Compare outliers and other unusual features
4. Compare shapes.

D. Exploring two-variable data sets

1. Scatterplots
2. Correlation and linear relationships
3. Least squares regression line
4. Residual plots, outliers, influential point
5. Logarithmic transformations and linearity.

II. Planning a Study

A. Overview of methods of data collection

1. Census
2. Sample survey
3. Experiment
4. Observational study

B. Planning and Conducting Surveys

1. Simple random sample
2. Sampling error
3. Well designed and conducted survey elements
4. bias
5. Stratified and Systematic sampling

C. Planning and Conducting Experiments

1. Experiments vs. Observational Studies
2. Confounding, control group, placebo, blinding
3. Treatment, experimental units, randomization
4. Randomized paired comparison design
5. Replication, blocking

III. Anticipating Patterns: Models Using Probability and Simulation

A. Probability

1. "Law of Large Numbers" concept
2. Addition Rule, Multiplication Rule, conditional probability, independence
3. Discrete random variables
4. Binomial, geometric distributions
5. Mean and Standard Deviation of random variable from 3,4.

B. Combining independent random variables

1. Notion of independent vs. dependent
2. Mean and Standard Deviation for sum and difference of independent random variables

C. Normal Distributions

1. Properties

2. Using tables of normal distributions
3. Using it as a model for measurements
- D. Sample Distribution
  1. Proportion Distribution
  2. Sample Mean Distribution
  3. Central Limit Theorem
  4. Difference between two independent sample proportions
  5. Difference between two independent sample means
- IV. Statistical Inference: Confirming Models
  - A. Confidence Intervals
    1. Meaning of
    2. For a proportion
    3. For a sample mean
    4. For a difference between two proportions
    5. For a difference between two means (paired or unpaired)
  - B. Significance Testing
    1. Null and Alternative Hypotheses, p-values, one and two sided tests
    2. Proportion Test
    3. Sample Mean Test
    4. Difference between Proportions
    5. Difference between Means (paired, unpaired)
    6. Chi-Square Testing for goodness of fit, independence
  - C. Special cases of normally distributed data
    1. t-distribution
    2. single sample t procedure
    3. Two sample (independent and matched pairs) t procedures
    4. Inference for slope of least squares line